 

# Second Exam MTH 221 , Fall 2011

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**QUESTION 1. ( Circle the correct answer, each = 2.5, total 17.5)**

(i) One of the following set is equal to $R^2$:

a) $Span\{(2,1),(6,3)\}$ b) $\{(3,0),(2,4)\}$ c) $\{(a+2b,-4b) \mid a,b \in R\}$ d) $Span\{(4,6)\}$

(ii) One of the following is a subspace of $R^3$:

a) $\{(a,2a,b^2) \mid a,b \in R\}$. b)$\{(a+b,0,2+b) \mid a,b \in R\}$ c) $\{(3,0,a+b) \mid a,b \in R\}$ d) $\{(a,3b-a,b) \mid a,b \in R\}$

(iii) Let $A$ be a particular matrix $3 \times 4$ such that $N(A) = \{(a_3, a_3+a_4, a_3, a_4) \mid a_3, a_4 \in R\}$. Then one of the following statement is true:

a) If the system of linear equations $AX = \begin{bmatrix} 2 \\ 3 \\ 4.2 \end{bmatrix}$ has a solution, then the solution is unique. b) There must be a point $B = (b_1, b_2, b_3) \in R^3$ such that the system $AX = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ has no solutions. c) The system $AX = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ has infinitely many solutions. d) None of the previous statements is correct.

(iv) Let $A$, $N(A)$ as in the previous question. Let $B$ be a matrix $4\times 3$ such that $Rank(B) = 2$ and $AB = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Then one of the following is a possibility for $B$:

a) $\begin{bmatrix} 1 & 3 & 0 \\ 1 & 3 & 2 \\ 1 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. b) $\begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ -1 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ d) It is possible that $B$ be as in (a) and as in (b) and as in (c). e) There is no way that we can determine a possibility for $B$ since $A$ is not completely determined.

(v) Let $A$, $N(A)$ as in (iii). Given that $A\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$. Then one of the following must be true:

a) $A\begin{bmatrix} 1 \\ 3 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ b) $A\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ c) $A\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ d) The solution to $AX = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ is unique and hence $x_1 = 0, x_2 = 1, x_3 = 1, x_4 = 1$ is the only solution to the system $AX = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

(vi) Let $F = Span\{(a+c)+(a+c)x+(a+b+2c)x^2 \mid a,b,c \in R\}$ be a subspace of $P_3$. Then $dim(F) =$

a) 2 b) 3 c)1 d) Cannot be determined

(vii) Let $F = \{(a+c, a+c, a+b+2c) \mid a,b,c \in R\}$. Then $F =$

$span\{(1,1,1),(0,0,6)\}$ b) $Span\{(2,2,2),(-1,-1,-1)\}$ c) Span $\{(1,1,2)\}$ d) $R^3$

**QUESTION 2. (Circle the correct answer, each = 2.5, total = 22.5)**

(i) Let $F = \{\begin{bmatrix} a-b & 0 \\ -a+b & 2a-2b \end{bmatrix} \mid a, b \in R\}$. Then $F =$

a) $Span\{\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}\}$ b) $Span\{\begin{bmatrix} 4 & 0 \\ -4 & 8 \end{bmatrix}\}$ c) $Span\{\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix}\}$ d) None of the previous statements

(ii) Let $F$ as above and $D \in F$ such that $D$ is not the zero matrix. Then Column space of D =

a) $R^2$ b) $R^4$ c) $Span\{(1,-1)\}$ d) $Span\{(1,0)\}$ e) Since different $D$ has different columns, column space of D cannot be determined.

(iii) Given $A$ is a $4 \times 4$ such that $A$ is row-equivalent to $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}$. Then $N(A) =$

a) $\{(x_2 + x_3, x_2, x_3, 0) \mid x_2, x_3 \in R\}$ b) $Span\{(1,1,1,1), (0,0,0,1)\}$ c) $Span\{(-1,1,0,0), (-1,0,1,0)\}$.
d) $\{(-x_2 + x_3, x_2, x_3, 0) \mid x_2, x_3 \in R\}$

(iv) Let $A$ as in the previous question. Then one of the following points DOES NOT belong to the row space of $A$

a) $(-1,-1,-1,5)$ b) (1, 1, 1, 0) c) (1, 1, -1, 4) d) (-1, -1, -1, 0)

(v) Let $A$ as in question (ii). Given $A \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 6 \end{bmatrix}$ and $A \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 4 \end{bmatrix}$. Then the column space of $A$ is

a) $span\{(0,0,0,2), (-1,0,0,1)\}$ b) $Span\{(1,0,0,0), (1,1,2,4)\}$ c) Span$\{(1,0,0,0), (0,1,0,0)\}$ d) span$\{(1,-1,-1,-1), (1,0,1,3)\}$ e) More information is needed to find Column space of $A$.

(vi) Let $F = \{f(x) \in P_3 \mid f(-1) = 0\}$. Then $F$ is a subspace of $P_3$. Hence $F =$

a) $Span\{6 + 6x, x^2 + 1\}$ b) span$\{x + x^2\}$ c) $Span\{x + x^2, 2x + x^2\}$ d) Span$\{x + 1, x^2 + 2x + 1\}$

(vii) One of the following is a basis for $R^4$

a) $\{(4,6,0,2), (-2,8,2,2), (-4,-6,3,7), (-2,-3,0,10)\}$ b) $\{(1,0,0,0), (1,1,0,1), (0,1,0,1), (0,0,0,4)\}$
C) Any 4 points in $R^4$ is a basis for $R^4$. d) (a) and (b) and (c) will do

(viii) One of the following points belong to $Span\{(1,1,1), (-1,1,1), (3,-1,-1)\}$

a) $(4, 2\pi, 2\pi^+2)$ b) (0, 2, 3) c) $(1, \pi, -\pi)$ d) (1, 6, 6)

(ix) Let $A$ be a $3 \times 3$ such that $(0,4,0) \in N(A)$, $A \begin{bmatrix} 1 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 10 \end{bmatrix}$, and $A \begin{bmatrix} 0 \\ 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ 10 \end{bmatrix}$. Then $N(A) =$

a) $Span\{0,1,0)\}$ b) $Span\{(0,4,0), (1,8,0), (0,7,1)\}$ c) $Span\{(0,4,0), (1,15,1)\}$ d)$span\{(0,1,0), (1,1,-1)\}$
e) More information is needed

**QUESTION 3. (JUST WRITE T OR F, Total = 10 points)**

(i) Let $F = \{A \in R^{2\times 2} \mid det(A) \leq 1\}$. Then $F$ is a subspace of $R^{2\times 2}$

(ii) The span of any 5 polynomials in $P_5$ is equal to $P_5$

(iii) $F = \{g(x) \in P_4 \mid f(0) = 0 \text{ or } f(1) = 0\}$ is a subspace of $P_4$

(iv) $D = \{f(x) \in P_3 \mid$f(0) = 1$\}$ is a subspace of $P_3$.

(v) It is possible to have 7 matrices in $R^{2\times 3}$ that are independent.

(vi) Every 6 points in $R^5$ are dependent

(vii) It is possible that the span of 6 points in $R^3$ is equal to $R^3$

(viii) If $A$ is $3 \times 3$ and the system $AX = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$ has infinitely many solution, then it is possible that $Rank(A) = 1$

(ix) If $A$ is a $3 \times 5$ matrix such that for every $B = (b_1, b_2, b_3) \in R^3$ the system $AX = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ has a solution, then dim(N(A)) = 2.

(x) $F = \{(b, -2a, 3 + b) \mid a, b \in R\}$ is a subspace of $R^3$

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